

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 948

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Unique Paper Code : 2352202002

Name of the Paper : Theory of Equations and Symmetries

Name of the Course : B.A. (Prog.) with Mathematics – DSC

Semester : III

Duration : 3 Hours

Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) Solve the equation  $x^3 - 13x^2 + 15x + 189 = 0$ , given that one of the roots exceeds another by 2.

P.T.O.

- (b) Find the nature of the roots of the equation, using Descartes's Rule of signs

$$x^4 - 2x^3 - 1 = 0.$$

- (c) Find a necessary condition for the roots of the equation  $x^3 - px^2 + qx - r = 0$  to be in harmonic progression.

2. (a) Using De Moivre's Theorem, show that

$$(1 + \cos\theta + i \sin\theta)^n + (1 + \cos\theta - i \sin\theta)^n = 2^{n+1} \cos^n(\theta/2) \cos n(\theta/2)$$

- (b) Find all the values of

$$\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$$

- (c) Solve the equation  $x^7 - 1 = 0$ .

3. (a) Solve the following cubic equation by Cardon's Method

$$y^3 - 9y + 28 = 0.$$

- (b) Find the equation whose roots are diminished by 3 the roots of  $x^4 - 7x^3 + 3x^2 - 11x + 17 = 0$ .

- (c) Solve the following biquadratic equation by Descartes Method

$$z^4 - 6z^2 - 16z - 15 = 0.$$

4. (a) Find the solution of equation  $y^3 - 15y - 126 = 0$  by Cardon's Method.

- (b) Find the equation whose roots are the reciprocals of the roots of the equation

$$x^6 + \frac{3}{4}x^5 - \frac{12}{5}x^4 + \frac{12}{5}x^2 - \frac{3}{4}x - 1 = 0.$$

- (c) Find the solution of equation  $z^4 + 3z^2 + 2z + 12 = 0$  by Descartes Method.

5. (a) Find the equation whose roots are 6 times the roots of  $x^3 + 3x^2 - 8x + 5 = 0$ .

- (b) If  $\alpha$ ,  $\beta$  and  $\gamma$  are the roots of the equation  $x^3 + 2x^2 - 3x - 1 = 0$ , then find the value of  $\alpha^{-3} + \beta^{-3} + \gamma^{-3}$ .

- (c) Find an equation whose roots are the reciprocals of the roots of the equation

$$x^4 - 3x^3 + 7x^2 - 8x + 2 = 0.$$

P.T.O.



6. (a) If  $\alpha$ ,  $\beta$  and  $\gamma$  are the roots of the equation  $x^3 - px^2 + qx - r = 0$ . Find

(i)  $\sum \alpha^2$

(ii)  $\sum \alpha^2 \beta$

- (b) If  $\alpha$ ,  $\beta$ ,  $\gamma$  are the roots of  $x^3 + qx + r = 0$ , find the value of  $\sum (\beta + \gamma)^2$ .

- (c) If  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  are the roots of  $x^4 + px^3 + qx^2 + rx + s = 0$ , find the value of

$$\sum \frac{1}{\alpha}$$